Longevity Risk and Precautionary demand for annuities

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Introduction

- “Full annuitization" result of Yaari (1965) without or with bequest/inter-vivos transfer...
- ...Due to the higher return delivered by an annuity contract compared to a bond (Davidoff, Brown, Yaari (2005))...
- ...Not to an insurance motive against mortality risk (idiosyncratic component of random duration of life).
- Indeed, in the "classical" expected lifetime utility setup, the individual is risk neutral with respect to uncertain duration of life.
- As pointed out by Bommier and Le Grand (2013), an early death (a very catastrophic event) can be (*ex ante*) fully compensated by higher future consumption if alive allowed by annuities, increasing the gap between the "good" and the "bad" states of the world (more risk).
- These two authors introduce an (non-additive) alternative set-up in which this gap matters. This reduces the demand for annuities, which become risky.

Introduction

- What happens if we introduce a so-called longevity risk, a systematic component of uncertain duration of life ? Two main questions in this (very) preliminary contribution.
- How does an uncertain survival probability modify the demand for annuities ?
- What kind of annuities is demanded by the individual in such a case ? What is the best longevity risk sharing scheme for an individual ?
- Simple Annuities Contract (SAC) with a return ex-ante guaranteed by a mortality table ? Or Group Self Annuitization (GSA) contract with a stochastic return depending on the realized survival probability ?
- In reality, a combination of these two contracts can be obtained by an appropriate choice of the Assumed Interest Rate (AIR) (*taux de rendement technique*) of the annuity contract.
- From both a theoretical and empirical point of view, the literature shows that the individual willingness to pay for a protection againt longevity risk is low, and this is related to two annuity puzzles :
- Low demand for annuities
- Attractiveness of GSA contract against SAC (see Boon, Brière and Werker (2017)).
- How a good new (a higher survival probability) affects current consumtion and the demand for annuities, especially in the GSA case where the return depends on the realized survival probability ?

Basic framework, 2 periods, exogenous saving

- 2 periods $= 1, 2$; $t = 1, 2, 3$; one individual with one heir ; exogenous saving $s > 0$.
- Mortality-longevity risks, uncertain duration of life at time 0 :

$$\delta = \begin{cases} 2 & \text{alive (a)} \quad \text{with proba. } \pi \\ 1 & \text{dead (d)} \quad \text{with proba. } 1 - \pi \end{cases}$$

- Uncertain lifetime "felicity" V, depending on the second period status (dead (d) or alive (a)) :

$$\begin{aligned} V_a(c, \tau_2) &= u_2(c) + v(\tau_2), \quad (\delta = 2) \\ V_d(\tau_1) &= v(\tau_1), \quad (\delta = 1) \end{aligned}$$

  with $c \geq 0$ consumption when alive in period 2, $\tau_2 \geq 0$ inter-vivos transfers when alive, $\tau_1 \geq 0$ bequests in case of death. $u_2(.)$ and $v(.)$ are increasing and concave functions with $u_2(0) > 0$.
- Portfolio choice between annuities and bonds :

$$s = a + b,$$

  with $b \geq 0$.
- No financial risk ; $R > 1$: return of the riskless bond ; R_a : return of the annuity with $R_a \geq R$.

Preferences toward mortality/longevity risks

- Lifetime utility = expected felicity with respect to life duration :

$$V(c, \tau_1, \tau_2) = \pi V_a(c, \tau_2) + (1 - \pi) V_d(\tau_1).$$

- Risk neutrality toward lifetime felicity risks, *i.e.* toward longevity/mortality risks : a linear transfer of felicity between the two status (dead or alive) has no consequence on total lifetime utility.
- Let us define the second period indirect utility function :

$$\mathcal{W}(Rs + (R_a - R)a) \equiv \max u_2(c) + v(\tau_2) \text{ subject to } c + \tau_2 \leq Rs + (R_a - R)a.$$

- By the Enveloppe theorem, one gets :

$$\mathcal{W}'(Rs + (R_a - R)a) = u_2'(c) = v'(\tau_2).$$

Fair annuity

- Assume fair annuity without loading $R_a = R/\pi$ and define lifetime utility :

$$\mathcal{V}(a,\pi) = \pi\mathcal{W}\left(R(s + \left(\frac{1-\pi}{\pi}\right)a\right) + (1-\pi)v(R(s-a))$$

- Optimal demand for annuities : $\max_a \mathcal{V}(a,\pi)$.
- "Classical case" ; full annuitization with altruism result (Yaari (1965), Davidoff, Brown and Diamond (2005)) :

$$R_a \leq \frac{R}{\pi} \Leftrightarrow c \geq R_a a \Leftrightarrow \tau_2 \leq Rb.$$

- Demand for annuities is motivated by returns dominance not by attitude toward risks. In case of fairness ($R_a = R/\pi$), one has $\tau_2 = \tau_1$ and $V_a > V_d$.
- Does it imply, that in this risk neutrality case, the individual can bear all the longevity risk ?

Mortality/Longevity risk and annuities

- Mortality risk : idiosyncrasic risk with a known survival probability :π.
- Longevity risk : survival probability π is random and distributed according to a known cumulative function $F(.)$. $\overline{\pi} = E(\pi)$.
- It does not change anything relative to preferences (see d'Albis and Thibault (2017) for a setup in which uncertain probabilities matter).
- Two kinds of annuities :
- SAC (Simple Annuity Contract), mortality and longevity are risk transfered to an annuity provider, such that the return on annuity is deterministic and based on a mortality table :
$$R_a = \frac{R}{F\overline{\pi}},$$
$F \geq 1$, the loading factor, is the risk premium paid to the annuity provider.
- GSA (Group Self Annuitization) contract in which an infinite number of individuals bear the systematic risks and pool the idosyncrasic ones. Annuities are risky with a stochastic return :
$$R_a = \frac{R}{\pi},$$
without any loading factor (no risk premium).

No loading factor

- Assume no loading factor ($F = 1$), then SAC always dominates GSA when π is stochastic :

$$\mathcal{V}_{SAC}(a) \geq \mathcal{V}_{GSA}(a)$$

- Proof : apply Jensen's law to the concave (w.r.t. to π) function :

$$\mathcal{V}(a, \pi) = \pi \mathcal{W}\left(R(s + \left(\frac{1-\pi}{\pi}\right) a\right) + (1-\pi) v(R(s-a))$$

- Even in a case of "risk neutrality", the individual prefers an unrisky return. No demand for collective annuities/GSA.
- SAC : Yaari "Classical" full annuitization result :

$$R_a \leq \frac{R}{\overline{\pi}} \Longleftrightarrow \tau_2 \leq \tau_1 = R(s-a) \Longleftrightarrow c \geq R_a a.$$

- No general result for the GSA case (it depends (in a complex way) on the third derivatives of the functions $u(.)$ and $v(.)$) such that we can not conclude on the sign of $a_{GSA} - a_{SAC}$.

Complete markets and contingent claims

- What is the result with a loading factor F ? This requires to think about market (in)completeness.
- As in Hanewald, Piggott and Sherris (2013), assume that the random survival probability π takes only two values such that for an individual, there are only 4 different states of the world, alive-dead (a, d) (idiosyncrasic component), (h, l) : high/low survival rate (systematic component).
- Conditional survival probabilities : $\pi(a|h) > \pi(a|l)$, we write : $\pi(a, h) = \pi(h)\pi(a|h), \ldots$

$$\overline{pi} = E\pi = \pi(a, h) + \pi(a, l) = \pi(h)\pi(a|h + (1 - \pi(h))\pi(a|l)$$

- Two definitions :
- Complete market : a contingent claim for each state of the world ; it pays 1 in the state and 0 in all other states. $p(a, h)$: price of the contingent claim paying 1 in the (a, h) state.
- When market is complete, the individual is fully able to choose at date 0 an optimal contingent consumption/transfer/bequest allocation : $(c(a, h), c(a, l), \tau_1(h), \tau_1(l), \tau_2(h), \tau_2(l))$ with only one (intertemporal) budget constraint.

Risk neutral pricing

- Pricing is risk neutral if for all prices :

$$p(a,h) = \pi(a,h)/R, ...$$

.

- When 1) market is complete, 2) pricing is risk neutral, and 3) lifetime utiliy is additive, one gets the full annuitization result such that $c(a,h) = c(a,l) = c$ and $\tau_1(h) = \tau_1(l) = \tau_2(h) = \tau_2(l)$ which can be implemented with only two basic assets : a Simple Annuity Contract and a riskless bond.
- All others assets are redundant : no need for a longevity bond or a GSA contract.

Incomplete market with a loading factor

- What happens if we introduce a more realistic 3 assets structure with incomplete market (lack of a longevity bond) and non-risk neutral pricing due to a loading factor.
- A riskless bond paying 1 in all states of the world, with a price :

$$p(a,h) + p(a,l) + p(d,h) + p(d,l) = 1/R.$$

- A Simple Annuity Contract with a loading factor $F \geq 1$ paying $1/\overline{\pi}$ if alive, such that :

$$p(a,h) + p(a,l) = F\overline{\pi}/R.$$

- A Group Self Annuitization arrangement (without loading factor) paying $1/\pi(a|h)$ if the state is (a,h) and $1/p(a|l)$ if the state is (a,l), such that :

$$\frac{p(a,h)}{\pi(a|h)} + \frac{p(a,l)}{\pi(a|l)} = \frac{\overline{\pi}}{R}.$$

- In such a case, the lack of a longevity bond (only the asset with price $p(d,h) + p(d,l)$) is available such that τ_1 can not be made contingent to h or l.

Incomplete market with a loading factor

- We can determine the two contingent prices $p(a,h)$ and $p(a,l)$ and show unfairness of these prices :

$$F \geq 1 \Longleftrightarrow \frac{p(a,l)}{\pi(a,l)} \leq \frac{1}{R} \leq \frac{p(a,h)}{\pi(a,h)}.$$

- Using the foc : $\pi(a,h)u_2'(c(a,h) = \lambda p(a,h)$, this implies :

$$F \geq 1 \Longleftrightarrow c(a,l) \geq c(a,h).$$

- This consumption allocation is implemented through a positive holding of GSA contract, such that there is a positive demand for this category of annuities for a positive loading factor.

A 3 period model with endogenous saving and Deferred Annuities Contract

- 3 periods $= 0, 1, 2$; and $t = 0, 1, 2, 3$.
- Death is uncertain at time 2 (end of the period 1) with a survival probability is $\pi \in (0, 1)$. Life is certain at time 1 and death is certain at time 3.
- Longevity risk : survival probability π (between 1 and 2) is random at date 0 and is realized at time 1, such that at this time there only remains (idiosyncrasic) mortality risk.
- Saving is exogenous at time 0 ($s_0 \geq 0$) and endogenous at time 1.
- 3 kinds of asset (market completness ?) :
- A one period riskless bond yielding a total return R.
- A Simple Annuity Contract available at time 1, with a deterministic return (if alive at time 2) $R_1 = R/(F_1 \pi)$ where $F_1 \geq 1$ is the loading factor.
- An illiquid Deferred Annuity Contract (DAC) available at time 0, with a deterministic return (if alive a time 2) $R_0 = R^2/(F_0 \overline{\pi})$ or a stochastic return (GSA) $R_0 = R^2/(F_0 \pi)$, with $F_0 \geq 1$ the loading factor.
- Is there a demand for the DAC at time 0 ? How does consumption at date 1 depend on the realization of the probability survival π ?

Two stages resolution

- 1) Backward resolution starting from time 1 :

$$V_1(Rb_0, a_0, \pi, R_1, R_0) = \max u_1(c_1) + (1-\pi)v(Rb_1) + \pi\mathcal{W}(Rb_1 + R_1 a_1 + R_0 a_0),$$

  under the budget constraint : $c_1 + b_1 + a_1 \leq Rb_0$.
- 2) Portfolio choice at time 0 given the optimal choice at time 1 :

$$\max_{a_0, b_0} E_\pi V_1(Rb_0, a_0, \pi, R_1, R_0) \text{ s. t. } b_0 + a_0 \leq s_0.$$

- $F_1 = 1$, full annuitization result at time 2 such that, for a given (b_0, a_0) :

$$\begin{cases} u_2'(c_2) = v_2'(\tau_2) \\ \tau_2 = Rb_1 \\ c_2 = R_1 a_1 + R_0 a_0 \\ c_1 + b_1 + a_1 = Rb_0 \\ u_1'(c_1) = Ru_2'(c_2) \end{cases}$$

Two stages resolution

- We solve the previous system in two cases : without and with GSA annuity contract available at date 0.
- We prove the two following results, for a given portfolio choice (a_0, b_0) made at date 0 :
- In both cases (No-GSA or GSA), c_1 decreases with the realization of survival probability π.
- In the GSA case, for a given portfolio choice (a_0, b_0), c_1 is more sensitive (compared to No-GSA) to π.
- (a_0, b_0) hs to be made endogenous at time 0 for having a clear conclusion.

Conclusion

- What is the optimal consumption path of an individual facing a longevity risk, *i.e.* a stochastic process describing its probability of survival ? In which direction does its consumption move when its probability of survival increases ?
- One may suspect that consumption and probability of survival probability move in opposite directions.
- Depends on preferences toward uncertain lifetime, market completness and the price of corresponding insurance scheme (fair, or with a loading factor).
- Without longevity bonds, markets are incomplete. In such a case, how GSA versus SAC may help to implement a second best optimal consumption path ? What is the relative preference of the individual among these two alternatives ?
- How to design an attractive annuity contract ?
- I must confess that I need to work more to obtain answers !

Bommier Le Grand (2013)

- Bommier and Le Grand (2013) : risk aversion toward lifetime utility risk. Lifetime utility has to be the expectation of a *concave* transformation of lifetime felicity :

$$V(c, \tau_1, \tau_2) = \pi \Phi(V_a(c, \tau_2)) + (1 - \pi)\Phi(V_d(\tau_1)),$$

  where $\Phi(.)$ is a strictly concave and increasing function.
- In this case "annuities transfer resources from a bad (=dead) to good (=alive) states of the world and are, as such, risk increasing".
- This reduces the demand for annuities which now appear riskier than bonds. In case of fair annuities, full-annuitization is no longer optimal.
- The individual do not care only about the expected lifetime felicity but also with the gap between felicities in the good and bad states.

D'Albis, Thibault (2017)

- Ambiguity and uncertain probabilities distribution :

$$V(c, \tau_1, \tau_2) = \pi\Phi(V_a(c, \tau_2)) + (1 - \pi)\Phi(V_d(\tau_1)),$$

where $\Phi(.)$ is a strictly concave and increasing function.