Longevity Risk and Precautionary demand for annuities

Juin 2018: Workshop Angers

Jérôme GLACHANT

CES, Université de Paris 1 Panthéon Sorbonne

28 juin 2018
Introduction

▶ "Full annuitization" result of Yaari (1965) without or with bequest/inter-vivos transfer...
▶ ...Due to the higher return delivered by an annuity contract compared to a bond (Davidoff, Brown, Yaari (2005))...
▶ ...Not to an insurance motive against mortality risk (idiosyncratic component of random duration of life).
▶ Indeed, in the "classical" expected lifetime utility setup, the individual is risk neutral with respect to uncertain duration of life.
▶ As pointed out by Bommier and Le Grand (2013), an early death (a very catastrophic event) can be (ex ante) fully compensated by higher future consumption if alive allowed by annuities, increasing the gap between the "good" and the "bad" states of the world (more risk).
▶ These two authors introduce an (non-additive) alternative set-up in which this gap matters. This reduces the demand for annuities, which become risky.
Introduction

▶ What happens if we introduce a so-called longevity risk, a systematic component of uncertain duration of life? Two main questions in this (very) preliminary contribution.
▶ How does an uncertain survival probability modify the demand for annuities?
▶ What kind of annuities is demanded by the individual in such a case? What is the best longevity risk sharing scheme for an individual?
▶ Simple Annuities Contract (SAC) with a return ex-ante guaranteed by a mortality table? Or Group Self Annuitzation (GSA) contract with a stochastic return depending on the realized survival probability?
▶ In reality, a combination of these two contracts can be obtained by an appropriate choice of the Assumed Interest Rate (AIR) (*taux de rendement technique*) of the annuity contract.
▶ From both a theoretical and empirical point of view, the literature shows that the individual willingness to pay for a protection against longevity risk is low, and this is related to two annuity puzzles:
▶ Low demand for annuities
▶ Attractiveness of GSA contract against SAC (see Boon, Brière and Werker (2017)).
▶ How a good new (a higher survival probability) affects current consumption and the demand for annuities, especially in the GSA case where the return depends on the realized survival probability?
Basic framework, 2 periods, exogenous saving

- 2 periods = 1, 2; t = 1, 2, 3; one individual with one heir; exogenous saving s > 0.
- Mortality-longevity risks, uncertain duration of life at time 0:

\[ \delta = \begin{cases} 
2 & \text{alive (a) with proba. } \pi \\
1 & \text{dead (d) with proba. } 1 - \pi 
\end{cases} \]

- Uncertain lifetime “felicity” V, depending on the second period status (dead (d) or alive (a)):

\[
V_a(c, \tau_2) = u_2(c) + v(\tau_2), \quad (\delta = 2)
\]
\[
V_d(\tau_1) = v(\tau_1), \quad (\delta = 1)
\]

with \( c \geq 0 \) consumption when alive in period 2, \( \tau_2 \geq 0 \) inter-vivos transfers when alive, \( \tau_1 \geq 0 \) bequests in case of death. \( u_2(.) \) and \( v(.) \) are increasing and concave functions with \( u_2(0) > 0 \).

- Portfolio choice between annuities and bonds:

\[ s = a + b, \]

with \( b \geq 0 \).

- No financial risk; \( R > 1 \) : return of the riskless bond; \( R_a \) : return of the annuity with \( R_a \geq R \).
Preferences toward mortality/longevity risks

- Lifetime utility = expected felicity with respect to life duration:
  \[ V(c, \tau_1, \tau_2) = \pi V_a(c, \tau_2) + (1 - \pi) V_d(\tau_1). \]

- Risk neutrality toward lifetime felicity risks, i.e. toward longevity/mortality risks: a linear transfer of felicity between the two status (dead or alive) has no consequence on total lifetime utility.

- Let us define the second period indirect utility function:
  \[ \mathcal{W}(Rs + (R_a - R)a) \equiv \max u_2(c) + v(\tau_2) \text{ subject to } c + \tau_2 \leq Rs + (R_a - R)a. \]

- By the Enveloppe theorem, one gets:
  \[ \mathcal{W}'(Rs + (R_a - R)a) = u'_2(c) = v'(\tau_2). \]
Fair annuity

- Assume fair annuity without loading $R_a = R / \pi$ and define lifetime utility:

$$\mathcal{V}(a, \pi) = \pi \mathcal{W} \left( R(s + \left( \frac{1 - \pi}{\pi} \right) a) + (1 - \pi) \nu(R(s - a)) \right)$$

- Optimal demand for annuities: $\max_a \mathcal{V}(a, \pi)$.

- "Classical case"; full annuitization with altruism result (Yaari (1965), Davidoff, Brown and Diamond (2005)):

$$R_a \leq \frac{R}{\pi} \Leftrightarrow c \geq R_a a \Leftrightarrow \tau_2 \leq R b.$$ 

- Demand for annuities is motivated by returns dominance not by attitude toward risks. In case of fairness ($R_a = R / \pi$), one has $\tau_2 = \tau_1$ and $V_a > V_d$.

- Does it imply, that in this risk neutrality case, the individual can bear all the longevity risk?
Mortality/Longevity risk and annuities

- Mortality risk: idiosyncratic risk with a known survival probability $\pi$.
- Longevity risk: survival probability $\pi$ is random and distributed according to a known cumulative function $F(.)$. $\overline{\pi} = E(\pi)$.
- It does not change anything relative to preferences (see d’Albis and Thibault (2017) for a setup in which uncertain probabilities matter).
- Two kinds of annuities:
  - SAC (Simple Annuity Contract), mortality and longevity are risk transferred to an annuity provider, such that the return on annuity is deterministic and based on a mortality table:
    \[ R_a = \frac{R}{F\overline{\pi}}, \]
    $F \geq 1$, the loading factor, is the risk premium paid to the annuity provider.
  - GSA (Group Self Annuitization) contract in which an infinite number of individuals bear the systematic risks and pool the idiosyncratic ones. Annuities are risky with a stochastic return:
    \[ R_a = \frac{R}{\pi}, \]
    without any loading factor (no risk premium).
Assume no loading factor \((F = 1)\), then SAC always dominates GSA when \(\pi\) is stochastic:

\[
V_{SAC}(a) \geq V_{GSA}(a)
\]

**Proof:** apply Jensen’s law to the concave (w.r.t. to \(\pi\)) function:

\[
V(a, \pi) = \pi W \left( R(s + \left(\frac{1 - \pi}{\pi}\right) a) + (1 - \pi)v(R(s - a)) \right)
\]

Even in a case of "risk neutrality", the individual prefers an unrisky return. No demand for collective annuities/GSA.

**SAC : Yaari "Classical" full annuitization result:**

\[
R_a \leq \frac{R}{\pi} \iff \tau_2 \leq \tau_1 = R(s - a) \iff c \geq R_a a.
\]

No general result for the GSA case (it depends (in a complex way) on the third derivatives of the functions \(u(.)\) and \(v(.)\)) such that we can not conclude on the sign of \(a_{GSA} - a_{SAC}\).
Complete markets and contingent claims

- What is the result with a loading factor $F$? This requires to think about market (in)completeness.
- As in Hanewald, Piggott and Sherris (2013), assume that the random survival probability $\pi$ takes only two values such that for an individual, there are only 4 different states of the world, alive-dead $(a, d)$ (idiosyncratic component), $(h, l)$: high/low survival rate (systematic component).
- Conditional survival probabilities: $\pi(a|h) > \pi(a|l)$, we write: $\pi(a, h) = \pi(h)\pi(a|h)$, ...
\[
\bar{p_i} = E\pi = \pi(a, h) + \pi(a, l) = \pi(h)\pi(a|h + (1 - \pi(h))\pi(a|l)
\]
- Two definitions:
- Complete market: a contingent claim for each state of the world; it pays 1 in the state and 0 in all other states. $p(a, h)$: price of the contingent claim paying 1 in the $(a, h)$ state.
- When market is complete, the individual is fully able to choose at date 0 an optimal contingent consumption/transfer/bequest allocation: $(c(a, h), c(a, l), \tau_1(h), \tau_1(l), \tau_2(h), \tau_2(l))$ with only one (intertemporal) budget constraint.
Risk neutral pricing

- Pricing is risk neutral if for all prices:

\[ p(a, h) = \pi(a, h)/R, \ldots \]

- When 1) market is complete, 2) pricing is risk neutral, and 3) lifetime utility is additive, one gets the full annuitization result such that \( c(a, h) = c(a, l) = c \) and \( \tau_1(h) = \tau_1(l) = \tau_2(h) = \tau_2(l) \) which can be implemented with only two basic assets: a Simple Annuity Contract and a riskless bond.

- All other assets are redundant: no need for a longevity bond or a GSA contract.
Incomplete market with a loading factor

- What happens if we introduce a more realistic 3 assets structure with incomplete market (lack of a longevity bond) and non-risk neutral pricing due to a loading factor.
- A riskless bond paying 1 in all states of the world, with a price:
  \[ p(a, h) + p(a, l) + p(d, h) + p(d, l) = 1/R. \]
- A Simple Annuity Contract with a loading factor \( F \geq 1 \) paying \( 1/\pi \) if alive, such that:
  \[ p(a, h) + p(a, l) = F\pi/R. \]
- A Group Self Annuitization arrangement (without loading factor) paying \( 1/\pi(a|h) \) if the state is \((a, h)\) and \(1/p(a|l)\) if the state is \((a, l)\), such that:
  \[ \frac{p(a, h)}{\pi(a|h)} + \frac{p(a, l)}{\pi(a|l)} = \frac{\pi}{R}. \]
- In such a case, the lack of a longevity bond (only the asset with price \( p(d, h) + p(d, l) \)) is available such that \( \tau_1 \) can not be made contingent to \( h \) or \( l \).
Incomplete market with a loading factor

- We can determine the two contingent prices $p(a, h)$ and $p(a, l)$ and show unfairness of these prices:

$$F \geq 1 \iff \frac{p(a, l)}{\pi(a, l)} \leq \frac{1}{R} \leq \frac{p(a, h)}{\pi(a, h)}.$$

- Using the foc: $\pi(a, h)u_2'(c(a, h)) = \lambda p(a, h)$, this implies:

$$F \geq 1 \iff c(a, l) \geq c(a, h).$$

- This consumption allocation is implemented through a positive holding of GSA contract, such that there is a positive demand for this category of annuities for a positive loading factor.
A 3 period model with endogenous saving and Deferred Annuities Contract

- 3 periods = 0, 1, 2; and \( t = 0, 1, 2, 3 \).
- Death is uncertain at time 2 (end of the period 1) with a survival probability is \( \pi \in (0, 1) \). Life is certain at time 1 and death is certain at time 3.
- Longevity risk: survival probability \( \pi \) (between 1 and 2) is random at date 0 and is realized at time 1, such that at this time there only remains (idiosyncratic) mortality risk.
- Saving is exogenous at time 0 \( (s_0 \geq 0) \) and endogenous at time 1.
- 3 kinds of asset (market completeness?) :
  - A one period riskless bond yielding a total return \( R \).
  - A Simple Annuity Contract available at time 1, with a deterministic return (if alive at time 2) \( R_1 = R/(F_1 \pi) \) where \( F_1 \geq 1 \) is the loading factor.
  - An illiquid Deferred Annuity Contract (DAC) available at time 0, with a deterministic return (if alive at time 2) \( R_0 = R^2/(F_0 \pi) \) or a stochastic return (GSA) \( R_0 = R^2/(F_0 \pi) \), with \( F_0 \geq 1 \) the loading factor.
- Is there a demand for the DAC at time 0? How does consumption at date 1 depend on the realization of the probability survival \( \pi \)?
Two stages resolution

- 1) Backward resolution starting from time 1:

\[ V_1(Rb_0, a_0, \pi, R_1, R_0) = \max u_1(c_1) + (1 - \pi)v(Rb_1) + \pi W(Rb_1 + R_1a_1 + R_0a_0), \]

under the budget constraint: \( c_1 + b_1 + a_1 \leq Rb_0. \)

- 2) Portfolio choice at time 0 given the optimal choice at time 1:

\[ \max_{a_0,b_0} E_\pi V_1(Rb_0, a_0, \pi, R_1, R_0) \text{ s.t. } b_0 + a_0 \leq s_0. \]

- \( F_1 = 1, \) full annuitization result at time 2 such that, for a given \((b_0, a_0)\):

\[
\begin{align*}
& u'_2(c_2) = v'_2(\tau_2) \\
& \tau_2 = Rb_1 \\
& c_2 = R_1a_1 + R_0a_0 \\
& c_1 + b_1 + a_1 = Rb_0 \\
& u'_1(c_1) = Ru'_2(c_2)
\end{align*}
\]
Two stages resolution

- We solve the previous system in two cases: without and with GSA annuity contract available at date 0.
- We prove the two following results, for a given portfolio choice \((a_0, b_0)\) made at date 0:
  - In both cases (No-GSA or GSA), \(c_1\) decreases with the realization of survival probability \(\pi\).
  - In the GSA case, for a given portfolio choice \((a_0, b_0)\), \(c_1\) is more sensitive (compared to No-GSA) to \(\pi\).
  - \((a_0, b_0)\) has to be made endogenous at time 0 for having a clear conclusion.
Conclusion

What is the optimal consumption path of an individual facing a longevity risk, i.e. a stochastic process describing its probability of survival? In which direction does its consumption move when its probability of survival increases?

One may suspect that consumption and probability of survival probability move in opposite directions.

Depends on preferences toward uncertain lifetime, market completeness and the price of corresponding insurance scheme (fair, or with a loading factor).

Without longevity bonds, markets are incomplete. In such a case, how GSA versus SAC may help to implement a second best optimal consumption path? What is the relative preference of the individual among these two alternatives?

How to design an attractive annuity contract?

I must confess that I need to work more to obtain answers!
Bommier and Le Grand (2013): risk aversion toward lifetime utility risk. Lifetime utility has to be the expectation of a concave transformation of lifetime felicity:

\[ V(c, \tau_1, \tau_2) = \pi \Phi(V_a(c, \tau_2)) + (1 - \pi) \Phi(V_d(\tau_1)), \]

where \( \Phi(.) \) is a strictly concave and increasing function.

In this case "annuities transfer resources from a bad (=dead) to good (=alive) states of the world and are, as such, risk increasing".

This reduces the demand for annuities which now appear riskier than bonds. In case of fair annuities, full-annuitization is no longer optimal.

The individual do not care only about the expected lifetime felicity but also with the gap between felicities in the good and bad states.
D’Albis, Thibault (2017)

- Ambiguity and uncertain probabilities distribution:

\[ V(c, \tau_1, \tau_2) = \pi \Phi(V_a(c, \tau_2)) + (1 - \pi) \Phi(V_d(\tau_1)), \]

where \( \Phi(.) \) is a strictly concave and increasing function.