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Why do rational people vote in large elections with costs to vote?

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Résumé : La plupart des gens votent lors d'élections nationales alors qu'il y a un coût pour voter et surtout que le bénéfice espéré est infinitésimal en comparaison, d'où le paradoxe. Nous montrons que les théories des perspectives et du regret ne peuvent résoudre ce paradoxe et peuvent même l'aggraver. Cependant, si la possibilité de produire un vote décisif vient à l'esprit, les gens maximisant leur espérance d'utilité vont douter de leur préférence première pour s'abstenir et surestimer la probabilité d'être décisif. Cela conduit à une solution rationnelle au paradoxe de l'électeur et, plus généralement, une solution rationnelle à l'illusion de contrôle.

Abstract: Many people vote in large elections with costs to vote although the expected benefits would seem to be infinitesimal to a rational mind. We show that prospect and regret theories cannot solve this paradox of not voting and may even aggravate it. However, if the possibility of a decisive vote comes to mind, expected utility maximizers will doubt their preference for abstention and greatly overestimate the decisiveness of their own ballot. This yields a canonical rational choice solution to the paradox of not voting and, more generally, a rationale for the illusion of control governing all sorts of magical acts.

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Why Do Rational People Vote in Large Elections with Costs to Vote?

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Abstract. Many people vote in large elections with costs to vote although the expected benefits would seem to be infinitesimal to a rational mind. We show that prospect and regret theories cannot solve this paradox of not voting and may even aggravate it. However, if the possibility of a decisive vote comes to mind, expected utility maximizers will doubt their preference for abstention and greatly overestimate the decisiveness of their own ballot. This yields a canonical rational choice solution to the paradox of not voting and, more generally, a rationale for the illusion of control governing all sorts of magical acts.

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1. Introduction

The expected utility from the act of voting in a large election is infinitesimal because the probability of a single vote being decisive (or pivotal) is infinitesimal. Consequently, the rational voter hypothesis first developed by Downs (1957) has been unable to explain why *rational* people vote to bring about the victory of their preferred candidate if the act of voting has a positive cost[1]: the expected benefit from voting would be of a smaller order of magnitude than the cost. People should never vote on rational grounds unless they have a taste for casting their ballot into an urn. However, few citizens never vote even though many occasionally abstain. This is fortunate for democracy and constitutes the well-known 'paradox of not voting' (PNV).

In his excellent review, Feddersen (2004) notes that previous attempts to solve this paradox have concentrated on the game-theoretic approach by lack of a canonical rational choice model of voting in elections with costs to vote. However, embedding the decision to vote within a game (Ledyard 1984, Palfrey and Rosenthal 1983) has not yielded a convincing solution to the PNV so far when uncertainty about the actual number of voters is introduced (Palfrey and Rosenthal 1985), or even when voters are structured in groups of supporters of the candidates who will cast their ballot if and only if they receive a consumption benefit from doing so (see Feddersen 2004 for references).

This paper revisits the decision-theoretic approach to the PNV by considering whether it can be explained in a non-expected utility framework[2]. This is a rather natural idea since expected utility (EU) has raised many other paradoxes of decision under risk and uncertainty, like Allais and Ellsberg paradoxes, which can be solved by non-EU (NEU) theories (see Starmer 2000) for a good review of this literature). Two prominent NEU theories which have fared well in other risky contexts are prospect theory (Kahneman and Tversky 1979) and rank-dependent EU (Quiggin 1982) on one hand, or regret theory (Bell 1982; Loomes and Sudgen

1982) on the other hand. For our present purpose, the distinguishing feature of these theories is that prospect theory and rank-dependent EU transform probabilities while regret theory modifies the utility function. Is the PNV solved by any of these theories? As we shall see, the answer is negative.

Therefore, we re-examine the decision to vote or abstain and establish two necessary conditions that a theory of rational decision under uncertainty must verify in order to solve the PNV. First, citizens must perceive that they share a responsibility in the electoral outcome: voting is an act under individual control which can turn defeat into victory. This is a *framing* condition. Second, citizens must substantially overweight the probability of victory for their preferred candidate. This implies *probability transformation*.

We give a rational solution to the PNV, obtained with cognitive consistency theory (Lévy-Garboua 1999; Blondel and Lévy-Garboua 2006), which verifies these two conditions while other prominent NEU theories, like prospect and regret theory, do not.

The paper proceeds as follows. We elicit necessary conditions for voting in section 2. Section 3 shows that prospect and regret theories cannot solve the PNV because they don't respect these two conditions together, and section 4 derives a rational solution which does. The latter is consistent with the civic duty motive for voting. Computations of turnout and winner's score are given in section 5 to illustrate the plausibility of our solution. Finally, it is briefly argued in section 6 that the PNV is an instance of a more general paradox known in psychology as the "illusion of control". Voting is, like praying and other magical actions, an act whereby people perceive that they have some control of a desired outcome of which they have objectively none. Concluding remarks appear in section 7.

2. Necessary conditions for voting with costs to vote

Let us briefly set up the notations that will be used here. The act of voting in a two-candidate election is viewed as a rational individual choice under uncertainty. It will be assumed throughout that individual voters have no power to form coalitions, an assumption that can be taken as a definition of a "large" election. This is the "one vote-one voice" motto of democracy. If the individual votes, he bears a cost, noted C in utility terms. We set up a level B (benefit) for the difference in utilities from the policies of the two candidates. The individual may vote (V) or abstain (A), and his preferred candidate can be elected or not. This leads to four possible utility levels:

[Insert Table 1 here]

C is positive, but typically small in comparison with the benefit from winning the election, so that:

$$-C < 0 < B - C < B \tag{1}$$

Obviously, if the decision to vote had no influence whatsoever on the electoral outcome, the individual would never vote if $C > 0$ because A would then strictly dominate V. This remark goes beyond the acceptance of EU rule because dominance is a universal criterion of rational choice. Thus we conclude that rational citizens who bear a cost of voting must perceive a positive probability of casting a decisive ballot in order to decide to vote. Voting is an act under individual control which can turn defeat into victory. Consequently, the decision problem is better described by table 2, which assumes that three states of the world are distinctly perceived. The individual's act of voting or abstention has no influence on the electoral outcome in state 1 (victory) and in state 2 (defeat), but it is decisive in the third state (voting is responsible for victory and abstention is responsible for defeat). Briefly stated, A does not quite dominate V. Potential voters will reason: 'If I do not vote, I can always save the cost of voting C but there is a possibility that I loose the (much) greater benefit of victory $B > C$ if my vote were to be decisive'. The argument stating that each voter might be decisive must be quite persuasive since it is commonly found that rather large majorities of voters do vote.

[Insert Table 2 here]

Let us now focus on an individual who evaluates the probabilities of the three states represented in table 2. Given all other votes, a single vote will change the electoral outcome if and only if either one of the two conditions below occurs:

(1) All other ballots are evenly split between the candidates (probability ε_1) so that an additional vote determines the winner. This may exclusively occur when there are an odd number of voters;

(2) All other ballots give victory to one's less preferred candidate by a margin of one vote. An additional vote for this candidate determines a draw, and the electoral outcome is eventually decided by an arbitrary criterion (probability $\varepsilon_2 \approx \frac{1}{2}\varepsilon_1$). This may exclusively occur when there is an even number of voters.

We can use $\varepsilon = \frac{1}{2}\varepsilon_1 + \frac{1}{2}\varepsilon_2 \approx \frac{3}{4}\varepsilon_1$ to assess the subjective probability that an individual's vote decide the victory of his preferred candidate (Mueller 1989, chap. 18). Let p be the expected percentage score for the preferred candidate and N the number of voters who expressed a clear preference through their ballot. The probability that any single vote be decisive is very small indeed. For instance, by using Owen and Grofman's (1984) approximation formula, we get:

$$\varepsilon = 1.5 \frac{\exp\left(-2(N-1)(p-0.5)^2\right)}{\sqrt{2\pi(N-1)}} \qquad (2)$$

Figures 1 and 2 show the probability that a single vote be decisive, derived from this formula, for various values of N and p. The probability of a decisive vote ε declines as the number of voters N increases and as the expected score of the preferred candidate p deviates from 0.5. We observe in figure 1 that ε is a tiny figure in large elections and that most citizens should have no reason to vote unless competition is extremely tight ($p \approx 0.5$).

[Insert Figures 1 and 2 here]

Myerson (2000) gave an alternative expression of the probability a vote is pivotal. Fischer (1999) pointed out that the probability of being decisive is sensitive to the approximation formula being used. However, we do not enter into this debate because, whatever formulation is used, ε remains infinitesimal and strictly positive. This last point is essential to ensure that the third state will occur in table 2 and consequently that *A* will not strictly dominate *V*.

[Insert Table 3 here]

The forgone discussion leads us to represent the decision to vote or abstain by table 3. Let q designate the estimated probability for victory of the preferred candidate of an individual if the latter did not vote. By deciding to vote, he gains an opportunity to increase this probability of winning by ε. However, such reason for voting will convince an EU maximizer if and only if: $(q+\varepsilon)B - C > qB$, or

$$\varepsilon.B > C \tag{3}$$

EU theory fails to predict that most people vote because ε is infinitesimal in a large election, particularly so as q gets further away from one-half. Voters must greatly overestimate the decisiveness of their own vote.

To conclude the above discussion, any rational solution to the PNV implies two hypotheses:

(H1) Rational citizens who bear a cost of voting must perceive voting as an act under individual control which can turn defeat into victory. Thus the decision to vote or abstain is framed in table 3 as a choice among two acts with three (voter-specific) states of the world.

(H2) The perceived probability of casting a decisive ballot must be substantially overestimated in comparison with ε.

Clearly, EU fails on both accounts. States of the world are immaterial in its formulation, and probabilities are not distorted.

The main conclusions of our analysis were derived in the standard framework of a two-candidate election and under the assumption that votes are instrumental in bringing about the victory of the preferred candidate. These two assumptions do not fit the big variety of political institutions and voting motives. Fortunately, the necessary conditions for voting elicited in the present paper are far more flexible than what so restrictive assumptions suggest. These conditions can be stated as follows: (i) rational citizens who bear a cost of voting must perceive voting as an act under individual control which can turn defeat into victory; (ii) voters substantially overestimate the decisiveness of their own ballot. The terms “defeat”, “victory”, and “decisiveness” may refer to any sort of “preferred outcome”, merely implying that, beyond some threshold value of votes expressing such preferred outcome, an individual experiences a jump in his utility level. This general formulation encompasses multi-candidate or multi-party elections and non-instrumental votes. Supporters of a minority party do not bear illusions about the possibility that their preferred candidate be elected. However, they may legitimately consider that a given - lower than fifty percent- score would be a victory by allowing their preferred candidate to reach a threshold in political representation and power. Even if a voter were indifferent between two candidates or parties, he might vote because he felt that his vote were decisive for democracy. If he cared for democracy, he would stand to gain from his fellow citizens voting and stand to loose from their abstention. If we simply assume that democracy is lost when voter turnout falls below a certain level, the individual decision to vote can still be formally described by table 3 in which the words ‘victory’ and ‘defeat’ apply to the preservation of democracy. Hence, a rational solution to the PNV may encompass a taste for democracy and need not be limited to the case of purely instrumental votes.

3. Can prospect and regret theories solve the paradox?

In this section, we ask whether prospect theory and/or regret theory provide a solution to the PNV. Indeed, prospect theory assumes probability transformation with an overweighting of small probabilities (Kahneman and Tversky 1979), and regret theory suggests that independent prospects be framed as actions with common states of the world. Thus these prominent NEU theories verify either hypothesis (H1) or (H2) which is needed to solve the PNV.

3.1. Prospect or rank-dependent expected utility theory

We use indifferently cumulative prospect or rank-dependent EU (RDEU) theory (Tversky and Kahneman 1992, Quiggin 1982) which respect dominance and extend to elections with more than two candidates. Both theories transform probabilities by substituting rank-dependent decision weights (which add up to one) for the expected percentage score of a candidate. After ranking states from worst to best, each state is given a weight that depends on its ranking as well as its probability, so that two candidates with equal expected score but unequal rank will receive a different weight.

In the two-candidate election case studied in the previous section, the decision to vote will be simply determined by:

$$w(q+\varepsilon)B - C > w(q)B ,$$

or:

$$[w(q+\varepsilon) - w(q)].B > C \tag{4}$$

Since ε is infinitesimal in a large election, prospect theory solves the PNV if and only if $w(q+\varepsilon) - w(q)$ is of a higher order of magnitude than ε for all values of q in a non-empty interval. However, this requires a discontinuity of the weighting function $w(r)$ at all values of r in a non-empty set. Thus prospect theory or RDEU cannot solve the PNV. For example, the weighting function which is most widespread in the literature assumes overweighting of small probabilities and underweighting of large probabilities. It is illustrated by figure 3.

[Insert Figure 3 here]

If this probability weighting function were adopted, $w(r)$ would be continuous and might even increase more slowly than r in the vicinity of one-half, so that: $w(r+\varepsilon)-w(r)<\varepsilon$. The PNV would then be aggravated by the use of prospect theory. To appreciate the likelihood of such possibility, take the weighting function $w(r)$ suggested by Tversky and Kahneman (1992), that is, $w(r)=\frac{r^\eta}{\left(r^\eta+(1-r)^\eta\right)^{\frac{1}{\eta}}}$ with η=.61. In this case, the PNV would indeed be aggravated if r lied between .085 and .845. These values characterize most democratic elections among two candidates. The only cases in which RDEU might predict voting concern values of r very close to 0 or 1, which contradicts intuition and empirical findings that closeness of election has a weak but positive effect on voter turnout[3].

The reason why prospect theory and RDEU fail to solve the PNV is that they verify hypothesis H2, but not H1. The small probability of casting a decisive ballot is not significantly overestimated because casting a decisive ballot is not isolated as an act that may change defeat into victory. Instead, such possibility is aggregated with the much larger probability q because RDEU aggregates all states that yield a common utility and thus share an equal rank.

3.2. Regret theory

Regret theory (Bell 1982; Loomes and Sudgen 1982) contends that people, when making a risky decision, anticipate the regret, and conversely the rejoicing, that their choice might generate after the resolution of uncertainty. Thus EU is modified by the addition of a regret/rejoicing function which relates possible outcomes of the chosen action to outcomes of the non chosen action. The regret/rejoicing function R is strictly increasing in the absolute value of regret, positive for rejoicing and negative for regret, and such that $R(0)=0$. The expected regret from one choice is symmetric to the expected rejoicing from its alternative choice. Table 4 shows that an individual expects to experience regret C with probability $1-\varepsilon$

if he decides to vote and to experience regret *(B-C)* with probability ε if he decides to abstain. He maximises the sum of his EU (table 3) and expected regret/rejoicing (table 4).

[Insert Table 4 here]

According to regret theory, an individual decides to vote if and only if:

$$(\varepsilon.B - C) + (1-\varepsilon)[R(-C) - R(C)] + \varepsilon[R(B-C) - R(C-B)] > 0 \qquad (5)$$

The second term in brackets is always negative and the third term in brackets is always positive. Since only the latter is infinitesimal, the sum of the two bracketed terms must be negative in a large election with costs to vote. Hence, if it is not EU-rational to vote when ε is infinitesimal (*i.e.,* $\varepsilon B - C < 0$), it cannot be rational to vote for regret theory under the same condition. If (3) does not hold, (5) cannot hold *a fortiori*; and the PNV is aggravated in comparison with EU. What goes on here is that regret theory verifies hypothesis H1, but not H2. Although the options of voting and abstention are perceived as actions, the probability of a decisive vote ε is not overestimated.

4. A rational theory of the decision to vote with costs to vote

4.1 Antecedents

Prospect and regret theory verify either H1 or H2, but not both. Not surprisingly, the celebrated solution proposed by Ferejohn and Fiorina (1974) exactly meets the two conditions: voting and abstention are framed as actions, and the probability of casting a decisive ballot is substantially overestimated. This arises from their assumption that uncertainty about the decisiveness of one's vote is total so that individuals are unable to calculate probabilities and simply adopt Minimax-regret decisions that do not require probability judgments. Potential voters decide to vote if and only if the regret of bearing an avoidable cost C (state 1 and 2 if *V*) is smaller than the regret of feeling responsible for defeat $B - C$ (state 3 if *A*). Evidently, the calculation of regret implies the framing of a decisive vote as an action, as shown by table 4. Furthermore, the decision rule under total uncertainty implies that the two states are given equal weights, which greatly overestimates the likelihood of a decisive vote. In our view, Ferejohn and Fiorina (1974) capture essential components of a solution to the PNV. However, their solution still suffers from two major deficiencies. First, the decision to vote is not –and cannot be - embedded within a game. And, second, it is not a rational decision rule since the probability of state 3 is totally disregarded.

Bendor, Diermeier and Ting (2003) address the first criticism only and solve the paradox with a behavioural model in which all people repeatedly interact with one another through an ergodic process of learning by trial and error[4]. People increase (decrease) their propensity to vote when the realized outcome (resulting from the actions of all voters including theirs) stands higher (lower) than their aspirations. At the same time, people adjust their aspirations to experienced payoffs. Assuming that the winning faction is not larger than the losing one by more than one member, turnout is pulled up in this model, despite the cost to vote, by the response of losing shirkers who incriminate their own abstention and vote more frequently in the next round. This is an important insight of the model. However, it may be argued that the long run equilibrium thus described is a remote idealization of what goes on in any particular election because the initial parameters of the turnout game undergo exogenous shocks from one election to the next that will often overshadow their endogenous variation induced by learning.

4.2 A canonical model of the decision to vote

We have so far elicited a number of conditions to be met by a canonical model of the decision to vote. The main result of this paper is that such model can be found. Indeed, we consider in this subsection a new theory which meets the two necessary conditions H1 and H2, predicts turnout in one-shot elections and assumes rational behavior. Our model of the decision to vote is actually embedded within a more general theory of choice under risk or uncertainty, first introduced in Lévy-Garboua (1999), called “cognitive consistency theory” (CC). CC theory can solve many other anomalies of choice under risk or uncertainty with a single parameter in addition to risk aversion (for a general exposition of this theory of choice under risk or uncertainty, see Lévy-Garboua 1999; and Blondel and Lévy-Garboua 2006). The whole argument proceeds in two steps. The first intuition is that the prior preference for abstention raises *doubt* because not voting does not quite dominate voting. The possibility of one's vote being decisive is a visible objection to one's preference for abstention (see table 3) that causes cognitive dissonance: the prior cognition is a choice between A and V while the second cognition, focused on the objection state, is another choice between two sure outcomes 0 and $B-C$. Furthermore, people would prefer to abstain in the first choice, but vote in the second. The next step of the argument is that cognitive dissonance causes within a rational mind a feeling of uncertainty regarding his “true preference” because it is not rational to hold two different preferences with certainty (Lévy-Garboua and Blondel 2002; Lévy-Garboua 2004). Indeed, a rational decision-maker who successively perceives the probability of his vote being decisive to be, first ε, then 1, is inclined to revise his prior assessment of probabilities upward, which may ultimately bring him to refute his prior preference for abstention on the day of election.

To express this formally, we treat the decisiveness of one’s vote as random and describe it by a Bernoulli variable which takes value 1 if this occurs and value 0 otherwise. The single parameter of this random variable is its mean which defines the expected likelihood of this event. Subjective EU theory yields a known value ε for this mean. We extend subjective EU by assuming that the mean of the vote’s decisiveness is randomly distributed in the unit interval. We designate this stochastic variable by $\tilde{\varepsilon}$ and make two further assumptions: (i) observed values of this random variable are independent identically distributed; (ii) the prior distribution of $\tilde{\varepsilon}$ is Beta with two positive parameters (φ, ψ) and mean ε. The first reflects the unpredictability of new cognitions to rational individuals who make use of all available information and the second is a technical assumption. From (ii), we derive values of the prior mean:

$$\varepsilon = \frac{\varphi}{\varphi + \psi} \tag{6}$$

and posterior mean (see DeGroot 1970: chapter 9, for instance):

$$\begin{aligned} \varepsilon^* &= \frac{\varphi + 1}{\varphi + \psi + 1} \\ &= \frac{\varphi + \psi}{\varphi + \psi + 1}\varepsilon + \frac{1}{\varphi + \psi + 1}1 \\ &\equiv \mu\varepsilon + (1-\mu)1 \end{aligned} \tag{7}$$

where $\mu \equiv \dfrac{\varphi + \psi}{\varphi + \psi + 1}$ ($0 < \mu \leq 1$) is the prior's weight and $1-\mu$ the objection's weight[5]. These weights reflect the individual's confidence in these two cognitions. The posterior mean is the *perceived probability*. It is greater than ε if the objection's weight is positive.

Thus, a Bayesian individual weighs his prior expected likelihood of decisiveness and the objection that he subsequently perceives in proportion to his confidence in these two cognitions. From (7), we derive that the choice between *A* and *V* can be described by an expected utility function in which the perceived probability of winning $\pi*$ is a positive affine transformation of the prior probability of winning $\pi(\equiv q + \varepsilon)$:

$$\begin{aligned} \pi^* &= \mu\pi + 1 - \mu \\ &= \pi + (1-\mu)(1-\pi) \end{aligned} \tag{8}$$

There is a striking identity between this expression for the perceived probability of winning of a specific voter and Bendor, Diermeier and Ting (2003) expression for the propensity to vote of a specific voter. However, the role played in their analysis by losing shirkers' learning *ex post* from their defeat is played here by the perception *ex ante* that one's prior preference for shirking can be responsible for defeat. Thus we need not assume myopic learning or strongly bounded rationality in order to explain why we observe substantial turnout in large elections with costs to vote. A rational theory of the decision to vote with costs to vote is a definite possibility.

The decision to vote can be derived from equation (3) by a mere substitution of the perceived probability of decisiveness (7) for the prior probability:

$$[\mu.\varepsilon + (1-\mu).1]B > C \tag{9}$$

Since the objection's weight is commonly much larger than the infinitesimal probability of casting a decisive ballot (Blondel and Lévy-Garboua 2006 derive for the average objection's weight an estimate of 0.3 from pairwise choices among bets), this explains why so many people vote even though it is not worth the cost. Furthermore, the infinitesimal ε-term can be generally neglected in comparison with the other left-hand term in (9). This establishes that individuals eventually vote when their idiosyncratic ratio of cost to benefit of voting does not exceed their objection's weight:

$$1-\mu \geq \frac{C}{B} \tag{10}$$

Condition (10) determines the rate of participation with a fair approximation in large elections. At first glance, it looks like the EU model's condition (3), with the objection's weight merely substituting for the probability of casting a decisive ballot. However, in contrast with (3), condition (10) separates each individual decision to vote from those of all other citizens. In a large election, each individual decides to vote or abstain almost in isolation, that is, without trying to guess what others do.

4.3 On democratic values and civic duty

A key factor of the individual propensity to vote, which constitutes the left-hand side of condition (10), reflects the degree of confidence in the decisiveness of one's vote. Since democratic values seek to persuade citizens that this is the case, it is likely to be conditioned by the length of exposition to democratic values. For instance, a Bayesian voter, exposed to *m* external messages signalling that his vote would make a difference and no other message pointing in the opposite direction, will perceive the probability of being decisive as:

$$\varepsilon*(m) = \frac{\varphi+\psi}{\varphi+\psi+1+m}\varepsilon + \frac{1+m}{\varphi+\psi+1+m}1 \tag{11}$$

$$= \frac{\mu}{1+m(1-\mu)}\varepsilon + \frac{1+m}{1+m(1-\mu)}(1-\mu)$$

In comparison with (7), derived from (11) when $m = 0$, the infinitesimal component gets even smaller and the finite component gets larger as the number of democratic messages increases. The perceived probability of a decisive vote tends to 1 if $m \to +\infty$. Thus, this model helps understand why younger adults, who have been exposed to democratic values for a shorter time, tend to participate less than older ones; and why very poor people, who feel excluded from society and deprived from any political influence, are often little concerned by elections. Citizens from non democratic regimes newly converted to democracy will, somewhat surprisingly, after an initial state of euphoria, lack confidence in the decisiveness of their own ballot since they haven't been exposed repeatedly in the past to the 'democratic framing' that each vote matters. Hence, the lower turnout which has been observed in new democracies like Portugal (after 1975) or Poland (after1989) can be explained by our model, and it does not mean that citizens from these countries would like to revert to the old regime.

Our theory is consistent with the idea that people vote because they adhere to democratic values and thus believe that their own preference matters[6]. As their beliefs grow stronger, people feel more compelled to vote non- strategically, even if they don't expect great benefits from the victory of their preferred candidate. Such description of voters' behavior is a *prima facie* indication that some people feel it is their "duty" to vote (almost) regardless of the costs and benefits of their vote. For instance, if we defined "voters by duty" as citizens who would always decide to vote as long as $C/B \leq 0.25$, we would classify in this category all citizens with a confidence in democratic values no smaller than $1-\mu = 0.25$. Assuming (see section

5) that $1-\mu$ is continuously uniformly distributed in the population on the interval $[0,0.5]$, 50% of citizens would be classified as "voters by duty". Our analysis shows, however, that duty is consistent both with rational behavior and with positive costs to vote.

5. Computations of turnout and winner's score

In this section, we make use of rules (9) and (10) to compute participation rates and winner's scores predicted by the CC model. Rule (9) indicates that the decision to vote depends directly on 4 parameters: μ, ε, B and C. If C is null or negative, anyone votes. Consequently, we assume positive costs to vote and, without loss of generality, normalize C to 1. We are left with three independent parameters: confidence in democracy $1-\mu$, probability of a decisive vote ε, and cost-benefit ratio $\frac{C}{B}$ (or the inverse ratio).

We assume that B follows a continuous uniform distribution between 1 (see constraint (1)) and $2\overline{B}-1$ (hence the mean is $\overline{B}$>1) and that μ follows a continuous uniform distribution on the interval $\left[2\overline{\mu}-1,1\right]$[7]. This last assumption leads to a uniform distribution of the perceived probability $\varepsilon *$ on the interval $[\varepsilon, \varepsilon_M]$, with $\varepsilon_M \equiv \left(2\overline{\mu}-1\right)\varepsilon + 2\left(1-\overline{\mu}\right)$.

The left-hand member of (9) displays the product of two continuous uniform distributions, $\varepsilon *$ and B. The latter has been characterized by Glen, Leemis and Drew (2004)[7]. Hence, the probability of voting is the probability that the product of the perceived probability $\varepsilon *$ and the benefit from winning B be greater than 1:

$$\Pr(V) = \begin{cases} \dfrac{\varepsilon_M\left(2\overline{B}-1\right)-\ln\left(\varepsilon_M\left(2\overline{B}-1\right)\right)-1}{2\left(\varepsilon_M-\varepsilon\right)\left(\overline{B}-1\right)} & \text{if} \quad \varepsilon\left(2\overline{B}-1\right)<1 \\ \dfrac{\ln(\varepsilon)-\ln\left(\varepsilon_M\right)+\left(2\overline{B}-1\right)\left(\varepsilon_M-\varepsilon\right)}{2\left(\varepsilon_M-\varepsilon\right)\left(\overline{B}-1\right)} & \text{otherwise} \end{cases} \tag{12}$$

This analytical formula yields, for given values of the parameters N, p, $\bar{\mu}$ and $\bar{B}$, the mean turnout and standard deviation $\frac{\sqrt{N.\Pr(V).(1-\Pr(V))}}{N}$.

5.1 Turnout and the number of voters

Let us first examine how the size of the electorate N conditions turnout by setting μ to the plausible average value .75. The probability of being pivotal ε is determined by N and the expected score of the preferred candidate p, using the Owen and Grofman's (1984) formula (2). We consider the case of a competitive political market with two candidates and thus assume for the expected score of the preferred candidate: $p = 0.5$.

[Insert Table 5 here]

Table 5 shows turnout as a function of $\bar{B}$ and N. The participation rate initially follows a steep decline when the population size increases and converges to a stable limit. For large elections, the participation rate no longer depends upon the population size and increases with the average gain from winning. For $\bar{B} = 4$, which corresponds to the assumption of Bendor, Diermeier and Ting (2003), the computed turnout rate is 41.6%, about 10% below the result found by Bendor, Diermeier and Ting (figure 7, p. 275). Turnout increases to 64.1% for $\bar{B} = 8$, 73.3% for $\bar{B} = 12$, and 81.7% for $\bar{B} = 20$.

5.2 Turnout and majority size in large elections

In the sequel, we focus on large elections and omit the infinitesimal effect of ε. The two equal factions will be called conventionally Left (L) and Right (R). All members of faction L prefer the left-wing candidate and all members of faction R prefer the right-wing candidate. The simple assumption of equally-sized factions reflects a competitive political equilibrium *à la* Hotelling in which each political platform attracts an equal number *N* of supporters. Each faction member *ij* has two parameters μ_{ij} and B_{ij}, with $i = (L, R)$ and $j = (1,2,...,N)$, that follow two independent continuous uniform distributions of parameters (means) $\bar{\mu} \in (0.5, 1.0)$

and $\overline{B_i} = (2,3,...,6,8,10,12,20)$ respectively[8]. We assume that μ follows the same distribution for Left and Right, but benefits from winning the election differ and reflect the different degree of motivation of both electorates. Due to the symmetry of the problem, degrees of motivation of groups (L, R) can be described by ($\overline{B}_L, \overline{B}_R$) with the restriction: $\overline{B}_L \geq \overline{B}_R$.

Since the rate of turnout will critically depend on the perceived benefits of winning the election, which usually differs between the two electorates, we can compute from the simulated samples both the global turnout rate $\overline{T} = \frac{T_L + T_R}{2}$ and the winner's score and majority size $S_L = \frac{T_L}{T_L + T_R}$. The results of our simulations are displayed in table 6[9].

[Insert Table 6 here]

Computed participation rates lie between 9.5% and 81.7%, which looks quite plausible. However, these values could be diminished by moving the μ-distribution to the right. For instance, if this distribution was uniform in the interval $[0.8,1.0]$ instead of $[0.5,1.0]$, participation rates would lie between 0% and 62.4% only. The lesser plausibility of such values confirms our choice of the μ-distribution on the larger interval, which had been suggested by previous results derived from choices among bets (Blondel and Lévy-Garboua 2006).

[Insert Table 7 here]

Tables 6 and 7 show the great impact of differential motivation upon both turnout and majority size. For instance, if $\overline{B}_L = 10$ and the Left wins the majority, turnout still exhibits considerable variation, from a low of 39.4% when the Right suffers from a very low motivation ($\overline{B}_R = 2$) to 66.8% when the Right enjoys high motivation ($\overline{B}_R = 8$). An interesting observation concerns the size of majorities in democracies. In table 7, very large majorities, say of at least 58%, imply an especially low motivation of the defeated group

($\overline{B}_R \leq 5$) but they do not imply very high motivation of the winning group. However, narrow majorities (in the zone of uncertainty between 50 and 51%) result essentially from an (almost) equal motivation of the two rival groups. When differential motivation is held constant, increasing $\overline{B}_i$ equally for $i = (R, L)$ raises turnout but reduces the margin of victory. If the differential motivation can be roughly assumed to be constant across districts on an election day, this may explain why a negative correlation is usually found between turnout and the winner's score across districts in a nationwide two-candidate election (Mueller 2003: 308-320).

6. Voting, praying, and other magical actions: the illusion of control

Finally, since the true probability of a decisive vote is immaterial in the solution of PNV, our theory offers a rational solution to the "illusion of control" governing all sorts of magical acts which have no effect *per se* on the desired outcome. For instance, many people pray for recovery of a loved person from a fatal disease, or prefer to choose their lucky numbers by themselves in purchasing lottery tickets although winning numbers are known to be random (Langer 1975). In all such cases, people incur a small cost for an act which has an infinitesimal expected return. The French scientist and philosopher Blaise Pascal (1966) legitimated his own religious belief by a reasoned bet on the truth of its predicament. The democratic belief that each vote matters for victory is similar to the religious belief that good life matters for salvation. Both postulate a reasoned bet on the decisiveness of one's act, that is, a specific framing by which people perceive that, by voting, praying, or any other magical action, they gain control of a jump in their own utility level. Beliefs of this kind can be forever sustained by the mechanism of illusory correlation: if my preferred candidate wins, I can attribute part of his merits to my vote.

Psychologists Quattrone and Tversky (1984, 1988) seem to be in close analogy when they speak of the « voter's illusion » to explain the PNV. They assume that people think that their own decision to vote is correlated with the decisions by people like them (typically from the same faction), so they turn out thinking that this will "cause" people like them to vote as well.

7. Concluding remarks

In the American presidential election which opposed George W. Bush to Al Gore in 2000, the margin of votes between the two candidates was extremely narrow and votes in Florida were presented *ex post* as being decisive. A similar event had occurred in 1960 when John Kennedy took a short lead over Robert Nixon in Illinois. Tight elections of this kind have been observed in other democratic nations as well. For example, in Italy, Romano Prodi beat Silvio Berlusconi in 2006 by a tiny difference of 0.07%. These rare events entertain the democratic belief that each vote matters and can be decisive. Indeed, if the probability to be decisive is infinitesimal, it is not zero. This difference is essential in a democracy and it legitimates the framing of the decision to vote in a large election. Rational citizens who bear a cost of voting must perceive voting as an act under individual control which can turn defeat into victory. Moreover, they must substantially overweight the probability of casting a decisive ballot. The same conclusions hold insofar the victory of a citizen's preferred outcome generates a jump in his utility level, whether the preferred outcome is the victory of his preferred candidate, the preservation of democracy, or the attainment of a minimum score for a minority candidate.

We presented in this paper a rational choice model of voting in large elections with costs to vote that can accommodate these necessary conditions for voting while prominent NEU theories like prospect and regret theory cannot. Although the decision to vote seems logically inconsistent to the normative eye, it is cognitively consistent to voters who doubt their normative preference for abstention and feel unsure of their true preference.

Our model of the decision to vote is actually embedded within a more general theory of choice under risk or uncertainty, first introduced in Lévy-Garboua (1999), called "cognitive consistency theory" (CC). Since CC theory has been able to solve many other anomalies of choice under risk or uncertainty with a single parameter in addition to risk aversion (see Lévy-Garboua 1999; Blondel 2002; Lévy-Garboua and Blondel 2002; Blondel and Lévy-Garboua 2006), we feel confident that the present solution to the paradox of not voting is not an *ad hoc* decision-theoretic formulation but can offer a canonical rational solution to the paradox of not voting.

Notes

[1] Without denying the fact that people may enjoy some aspects of voting, we rule out assumptions of a negative cost of voting due to a taste for voting (Riker and Ordeshook 1968), or to a taste for participation to collective actions, as in expressive voting theory (e.g., Schuessler 2000). However, individual tastes are essentially heterogeneous and voting is only one way, not necessarily efficient, of expressing one's social identity. In this paper, we look for existence of a universal drive for voting in democratic elections.

[2] Chew and Konrad (1998) is an exception for calling upon uncertainty aversion to justify bandwagon effects on voting behaviour. However, they don't address the question of why people decide to vote.

[3] Mueller (2003) contains a survey of studies bearing on this point, both on aggregate and individual data.

[4] The process converges to a unique limiting distribution from any initial set of propensities and aspirations (see Proposition 1 in Bendor, Diermeier and Ting 2003).

[5] Note that EU is a special case when individuals feel perfectly sure of their prior preference, *i.e.*, $\mu = 1$.

[6] An experiment by Blais and Young (1999) confirms that an emphasis on the economic reasons for not voting during the 1993 Canadian federal election campaign had a negative influence on turnout for a group of students (and potential voters) by inhibiting their perception of the positive reasons for voting. Framing matters.

[7] When X and Y both follow two continuous uniform distributions, respectively between $[a,b]$ and $[1,c]$, with $0 < a < b < 1 < c$, and $\delta = 1/(b-a)(c-1)$, the probability distribution function of the product Z of X and Y is given by,:

$$If\ \ ac < b : f(z) = \begin{cases} \delta.\ln\left(\dfrac{z}{a}\right) & for \quad a \le z \le ac \\ \delta.\ln(c) & for \quad ac \le z \le b \\ \delta.\ln\left(\dfrac{bc}{z}\right) & for \quad b \le z \le bc \end{cases}$$

$$If\ \ ac \ge b : f(z) = \begin{cases} \delta.\ln\left(\dfrac{z}{a}\right) & for \quad a \le z \le b \\ \delta.\ln\left(\dfrac{b}{a}\right) & for \quad b \le z \le ac \\ \delta.\ln\left(\dfrac{bc}{z}\right) & for \quad ac \le z \le bc \end{cases}$$

In the voting decision, the parameters *a*, *b* and *c* take respectively values ε, $\left(2\overline{\mu}-1\right)\varepsilon + 2\left(1-\overline{\mu}\right)$ and $2\overline{B}-1$. Note that the formula above could be applied to any product of two continuous uniform distributions where one variable (here the benefit cost ratio B/C) is always greater than the other one (here the perceived probability $\varepsilon *$), simply by normalizing the minimum of the higher variable to 1.

Hence, the probability of voting is obtained by integrating $f(z)$ between 1 and bc. We have benefited from the precious help of John Drew and Andrew Glen who kindly communicated these expressions to us.

[8] μ_i is a uniform distribution on the positive interval $\left(2\overline{\mu}-1, 1\right)$ and B_i is a uniform distribution on $\left(1, 2\overline{B}-1\right)$. The assumption of uniformity, chosen for its computational tractability, requires: $\overline{\mu} \ge 1/2$ and $\overline{B} > 1$.

[9] The winner's score is normally higher than 50%. However, when the two factions are of equal size and equally motivated (*i.e.* $\overline{B}_L = \overline{B}_R$), the final winner is randomly determined. Under our convention, it is possible to have: $S_L < S_R$ by a small margin. This can be seen in table 7 at low degrees of motivation.

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TABLE 1. The four possible utility levels

	Preferred candidate wins	Preferred candidate looses
A	B	0
V	$B-C$	$-C$

TABLE 2. The choice between voting (V) or not voting (A)

	The vote is not decisive		The vote is decisive
	Victory: State 1	Defeat: State 2	State 3
A	B	0	0
V	$B-C$	$-C$	$B-C$

TABLE 3. The risky choice between voting (V) or not voting (A)

	$\Pr[\text{state } 1]=q$	$\Pr[\text{state } 2]=1-q-\varepsilon$	$\Pr[\text{state } 3]=\varepsilon$
A	B	0	0
V	$B-C$	$-C$	$B-C$

TABLE 4. The potential regret/rejoicing with voting (V) or not voting (A)

	$\Pr[state\,1 \text{ or } state\,2]=1-\varepsilon$	$\Pr[state\,3]=\varepsilon$
A	rejoicing: C	regret: $C-B$
V	regret: $-C$	rejoicing: $B-C$

TABLE 5. Variation of average turnout with population size for several values of the benefits from winning the election (standard deviation in parentheses) (%)

		N						
		10	100	1000	10 000	100 000	10 Millions	Infinite
$\overline{B}$	2	26.5 (13.9)	13.4 (3.4)	10.6 (1.0)	9.8 (.3)	9.6 (.1)	9.5 (0)	9.5 (0)
	4	70.8 (14.4)	49.6 (5.0)	44.0 (1.6)	42.3 (.5)	41.8 (.2)	41.6 (0)	41,6 (0)
	6	82.5 (12.0)	63.7 (4.8)	58.7 (1.6)	56.8 (.5)	56.2 (.2)	55.9 (0)	55.9 (0)
	8	87.5 (10.5)	74.7 (4.4)	67.1 (1.5)	65.0 (.5)	64.4 (.2)	64.1 (0)	64.1 (0)
	10	90.3 (9.4)	79.8 (4.0)	72.6 (1.4)	70.4 (.5)	69.7 (.2)	69.5 (0)	69.4 (0)
	12	92.0 (8.6)	83.5 (3.7)	76.5 (1.4)	74.3 (.4)	73.6 (.1)	73.3 (0)	73.3 (0
	20	95.4 (6.6)	90.4 (2.9)	85.2 (1.1)	82.8 (.4)	82.1 (.1)	81.8 (0)	81.7 (0)

Note: $\mu \in [0.5;1.0]$ and $B \in [1;\overline{2B}-1]$

TABLE 6. Average turnout and differential motivation with two heterogeneous factions of equal size (%).

		$\overline{B}_L$								
		2	3	4	5	6	8	10	12	20
$\overline{B}_R$	2	9.5								
	3	19.3	29.2							
	4	25.5	35.4	41.6						
	5	29.7	39.5	45.7	49.9					
	6	32.7	42.5	48.7	52.9	55.9				
	8	36.8	46.6	52.8	57.0	60.0	64.1			
	10	39.4	49.3	55.5	59.7	62.7	66.8	69.4		
	12	41.4	51.2	57.4	61.6	64.6	68.7	71.3	73.3	
	20	45.6	55.5	61.7	65.8	68.8	72.9	75.6	77.5	81.7

Notes: $\mu \in [0.5;1.0]$

TABLE 7. Average winner's score and differential motivation with two heterogeneous factions of equal (%)

		$\overline{B}_L$								
		2	3	4	5	6	8	10	12	20
$\overline{B}_R$	2	50.0								
	3	75.5	50.0							
	4	81.5	58.8	50.0						
	5	84.1	64.1	54.5	50.0					
	6	85.5	65.7	57.4	52.8	50.0				
	8	87.1	68.7	60.6	56.2	53.4	50.0			
	10	88.0	70.4	62.5	58.2	55.4	52.0	50.0		
	12	88.6	71.5	63.8	59.5	56.7	53.3	51.3	50.0	
	20	89.6	73.7	66.3	62.1	59.4	56.1	54.1	52.7	50.0

Notes: $\mu \in [0.5;1.0]$

FIGURE 1. The probability that a single vote be decisive in a large election

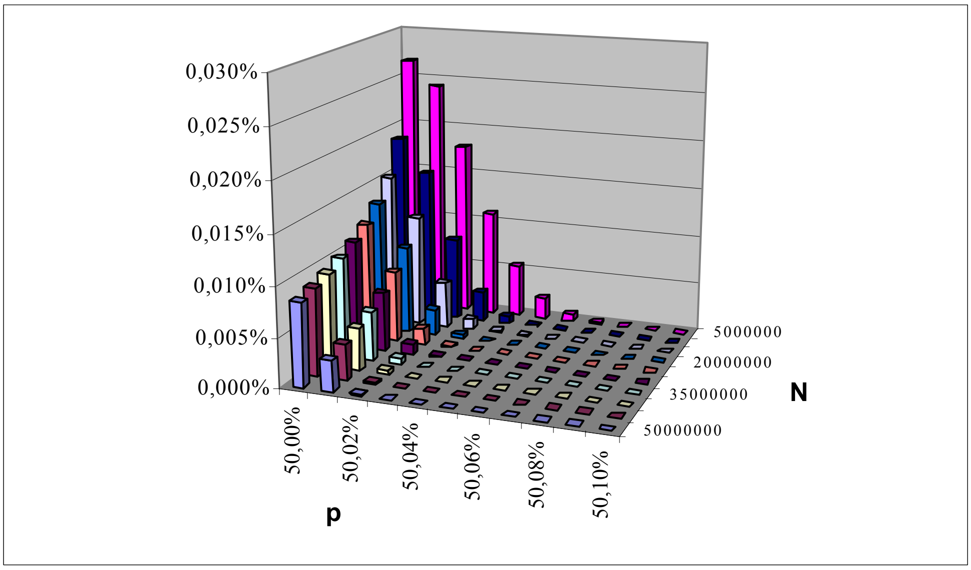


FIGURE 2. The probability that a single vote be decisive in a local election

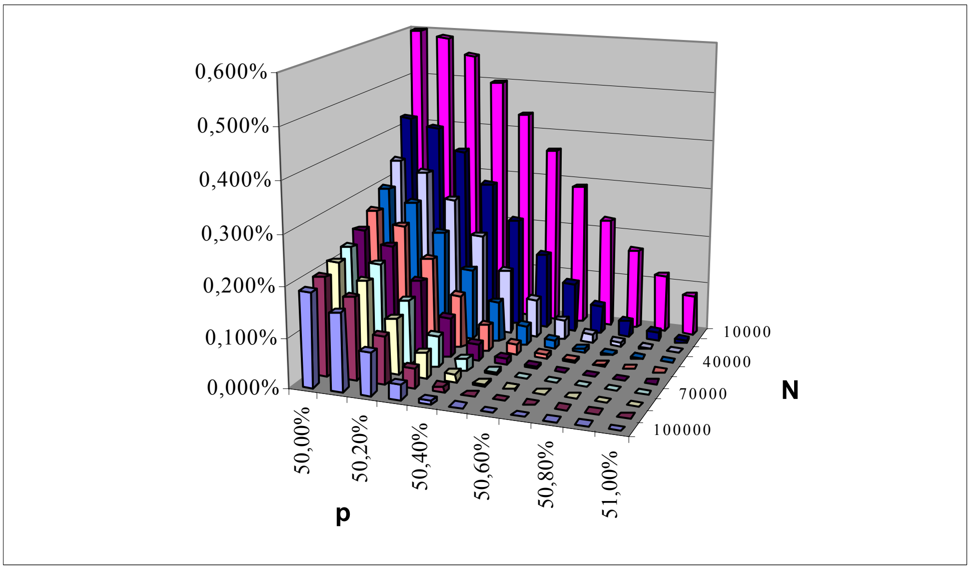
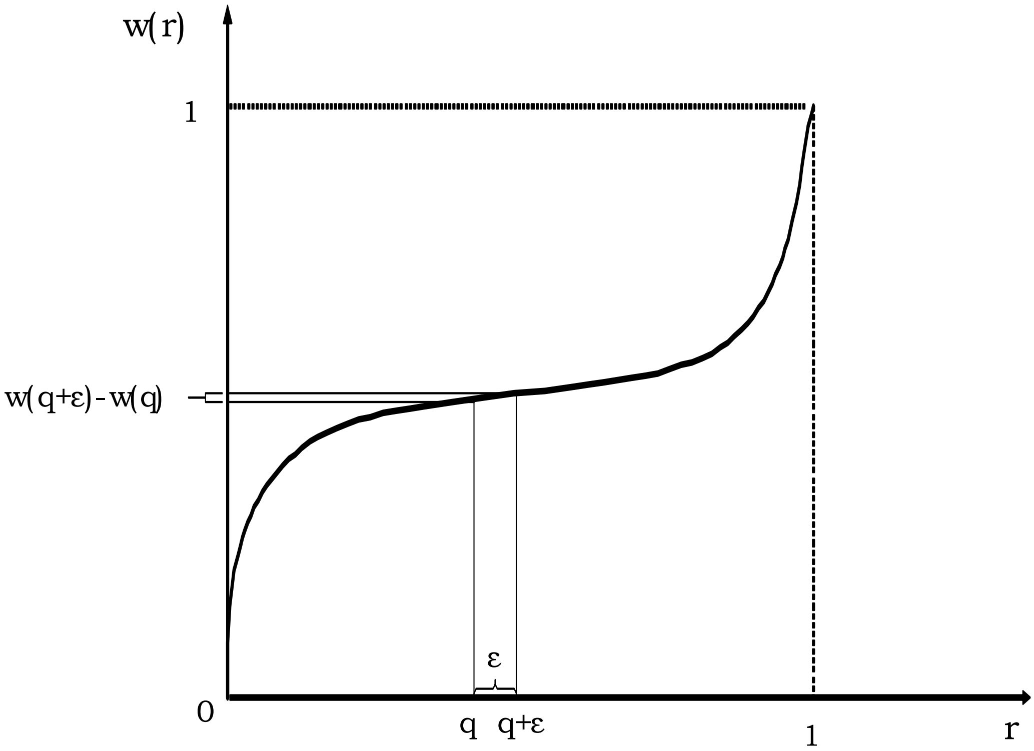


FIGURE 3. A weighting function



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